**ASSIGNMENT #3 Problem 1**

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**Consider the boundary value problem:**

a) Let , show that ϕ and ψ satisfy the differential equations:

* and

We will make the substitution into our homogeneous solution. The second derivatives are, and .

Bring terms to either side of the equal sign to get,

Isolate the x and y functions by division to get,

So we get two second order differential equations:

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b) Show that , , and .

Let us examine the first 2nd ODE from part a):

If we let , then , substitute this back into our differential equation to get,

This will yield imaginary solutions for λ > 0: . Therefore, our solution to the ϕ equation will be of the form:

Now we will use the boundary conditions:

If we substitute 0 into our ϕ function we get:

So our solution will be of the form:

If we utilize the second boundary condition we get,

So our eigen-functions are: .

Hence,

and

Now let us consider the second differential equation:

If we make a similar substitution we get,

Therefore, our solutions will be of the form: and . But recall the following identities for hyperbolic functions:

and

If we add the above two equations and subtract we get,

Therefore, our solutions to ψ are:

But remember linear combinations of our solutions will work as well so,

We already know λ:

So our solution for a particular *n* is:

By superposition we get,

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c) We will now find the coefficients for and . We will begin with the first initial condition:

We get,

Therefore we have the following series:

Hence,

Next we will let y = h into our function u(x,y) to help obtain the ‘d’ coefficients:

Notice that the entire term in parentheses is the coefficient for the Fourier Sine series expansion of g(x):

If we subtract and then divide we can isolate the ‘d’ coefficient:

This becomes,

for n = 1,2,3,4…

d) See mathematica File